Chapter Sixteen

Mensuration

The length of a line, the area of a place, the volume of a solid etc. are determined for practical purposes. In the case of measuring any such quantity, another quantity of the same kind having some definite magnitude is taken as unit. The ratio of the quantity measured and the unit defined in the above process is the amount of the quantity.

i.e. magnitude $= \dfrac{\text{Quantity measured}}{\text{Unit quantity}}$

In the case of a fixed unit, every magnitude is a number which denotes how many times of the unit of the magnitude is the magnitude of the quantity measured. For example, the bench is 5 metre long. Here metre is a definite length which is taken as a unit and in comparison to that the bench is 5 times in length.

At the end of the Chapter, the students will be able to –

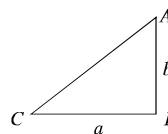
- Determine the area of polygonal region by applying the laws of area of triangle and quadrilateral and solve allied problems.
- Determine the circumference of the circle and a length of the chord of a circle.
- Determine the area of circle.
- Determie the area of circle. Determining the area of a circle and its part, solve the alied problems.
- Determine the area of solid rectangles, cubes and cylinder and solve the allied problems.
- Determine the area of uniform and non uniform polygonal regions.

16·1 Area of Triangular region

In the previous class, we learned that area of triagular region $= \frac{1}{2}$ base × height.

(1) **Right Angled Triangle :**

Let in the right angled triangle ABC, $BC = a$ and $AB = b$ are the adjacent sides of the right angle. Here if we consider BC the base and AB the height,

$$\text{Area of } \Delta ABC = \frac{1}{2} \text{ base} \times \text{height}$$
$$= \frac{1}{2}ab$$

(2) Two sides of a triangular region and the angle included between them are given.

Let in ΔABC the sides are :
$BC = a$, $CA = b$, $AB = c$.
AD is drawn perpendicular from A to BC.
Let altitude (height) $AD = h$.

Considering the angle C, we get, $\dfrac{AD}{CA} = \sin C$

or, $\dfrac{h}{b} = \sin C$ or, $h = b \sin C$

Area of $\Delta\ ABC = \dfrac{1}{2} BC \times AD$

$= \dfrac{1}{2} a \times b \sin C$

$= \dfrac{1}{2} ab \sin C$

Similarly, area of $\Delta\ ABC = \dfrac{1}{2} bc \sin A$

$= \dfrac{1}{2} ca \sin B$

(3) Three sides of a triangle are given.

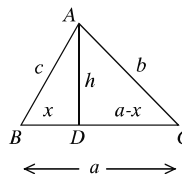
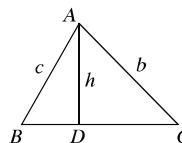
Let in ΔABC, $BC = a$, $CA = b$ and $AB = c$.
$\therefore$ Perimeter of the triangle $2s = a + b + c$
We draw $AD \perp BC$
Let, $BD = x$, so $CD = a - x$

In right angled ΔABD and ΔACD
$\therefore AD^2 = AB^2 - BD^2$ and $AD^2 = AC^2 - CD^2$
$\therefore AB^2 - BD^2 = AC^2 - CD^2$
or, $c^2 - x^2 = b^2 - (a - x)^2$
or, $c^2 - x^2 = b^2 - a^2 + 2ax - x^2$
or, $2ax = c^2 + a^2 - b^2$

$\therefore x = \dfrac{c^2 + a^2 - b^2}{2a}$

Again, $AD^2 = c^2 - x^2$

$= c^2 - \left(\dfrac{c^2 + a^2 - b^2}{2a}\right)^2$

$= \left(c + \dfrac{c^2 + a^2 - b^2}{2a}\right)\left(c - \dfrac{c^2 + a^2 - b^2}{2a}\right)$

$$= \frac{2ac + c^2 + a^2 - b^2}{2a} \cdot \frac{2ac - c^2 - a^2 + b^2}{2a}$$

$$= \frac{\{(c+a)^2 - b^2\}\{b^2 - (c-a)^2\}}{4a^2}$$

$$= \frac{(a+b+c)(a+b+c-2b)(a+b+c-2a)(a+b+c-2c)}{4a^2}$$

$$= \frac{2s(2s-2b)(2s-2a)(2s-2c)}{4a^2}$$

$$= \frac{4s(s-a)(s-b)(s-c)}{a^2}$$

$\therefore\ AD = \frac{2}{a}\sqrt{s(s-a)(s-b)(s-c)}$

Area of $\Delta\ ABC\ = \frac{1}{2} BC \cdot AD$

$$= \frac{1}{2} \cdot a \cdot \frac{2}{a}\sqrt{s(s-a)(s-b)(b-c)}$$

$$= \sqrt{s(s-a)(s-b)(b-c)}$$

(4) Equilateral Triangle :

Let the length of each side of the equilateral triangular region ABC be a.

Draw $AD \perp BC\ \therefore\ BD = CD = \frac{a}{2}$

In right angled ΔABD

$BD^2 + AD^2 = AB^2$

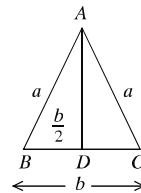
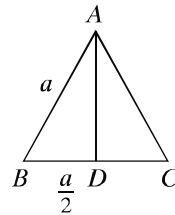
or, $AD^2 = AB^2 - BD^2 = a^2 - \left(\frac{a}{2}\right)^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4}$

$\therefore\ AD = \frac{\sqrt{3}a}{2}$

Area of $\Delta\ ABC\ = \frac{1}{2} \cdot BC \cdot AD$

$$= \frac{1}{2} \cdot a \cdot \frac{\sqrt{3}a}{2} \text{ or, } \frac{\sqrt{3}}{4}a^2$$

(5) Isosceles triangle :

Let ABC be an isosceles triangle in which $AB = AC = a$ and $BC = b$

Draw $AD \perp BC$. $\therefore BD = CD = \frac{b}{2}$

In ΔABD right angled

$\therefore AD^2 = AB^2 - BD^2 = a^2 - \left(\frac{b}{2}\right)^2 = a^2 - \frac{b^2}{4} = \frac{4a^2 - b^2}{4}$

$\therefore AD = \frac{\sqrt{4a^2 - b^2}}{2}$

Area of isosceles $\Delta ABC = \frac{1}{2} \cdot BC \cdot AD$

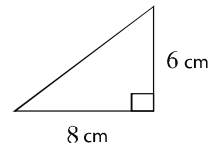
$= \frac{1}{2} \cdot b \cdot \frac{\sqrt{4a^2 - b^2}}{2}$

$= \frac{b}{4}\sqrt{4a^2 - b^2}$

Example 1. The lengths of the two sides of a right angled triangle, adjacent to right angle are 6 cm. and 8 cm. respectively. Find the area of the triangle.

Solution : Let, the sides adjacent to right angle are $a = 8$ cm. and $b = 6$ cm. respectively.

$\therefore$ Its area $= \frac{1}{2}ab$

$= \frac{1}{2} \times 8 \times 6$ square cm. = 24 square cm.

Required area 24 square cm.

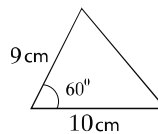
Example 2. The lengths of the two sides of a triangle are 9 cm. and 10 cm. respectively and the angle included between them is 60°. Find the area.

Solution : Let, the sides of triangle are $a = 9$ cm. and $b = 10$ cm. respectively.

Their included angle $\theta = 60^\circ$.

$\therefore$ Area of the triangle $= \frac{1}{2}ab\sin 60^o$

$= \frac{1}{2} \times 9 \times 10 \times \frac{\sqrt{3}}{2}$ sq. cm.

$= 38 \cdot 97$ sq. cm. (approx)

Required area $38 \cdot 97$ sq. cm. (approx)

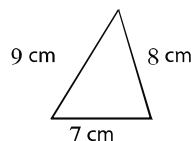
Exmple 3. The lengths of the three sides of a triangle are 7 cm., 8 cm. and 9 cm. respectively. Find its area.

Solution : Let, the lengths of the sides of the triangle are $a = 7$ cm., $b = 8$ cm. and $c = 9$ cm.

$\therefore$ Semi perimeter $s = \dfrac{a+b+c}{2} = \dfrac{7+8+9}{2}$ cm. = 12 cm.

$\therefore$ Its area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{12(12-7)(12-8)(12-9)}$ sq. cm.

$= \sqrt{12 \times 5 \times 6 \times 7}$ sq. cm. $= 50 \cdot 2$ sq. cm. (approx)

$\therefore$ The area of the triangle is $50 \cdot 2$ sq. cm. (approx)

Example 4. The area of an equilateral triangle increases by $3\sqrt{3}$ sq. metre when the length of each side increases by 1 metre. Find the length of the side of the triangle.

Solution : Let, the length of each side of the equilateral triangle is a metre.

$\therefore$ Its area $= \dfrac{\sqrt{3}}{4} a^2$ sq. m.

The area of the triangle when the length of each side increases by 1m. $= \dfrac{\sqrt{3}}{4}(a+1)^2$ sq. metre.

According to the question, $\dfrac{\sqrt{3}}{4}(a+1)^2 - \dfrac{\sqrt{3}}{4}a^2 = 3\sqrt{3}$

or, $(a+1)^2 - a^2 = 12$; [dividing by $\dfrac{\sqrt{3}}{4}$]

or, $a^2 + 2a + 1 - a^2 = 12$ or, $2a = 11$ or, $a = 5 \cdot 5$

The required length is $5 \cdot 5$ metre.

Example 5. The length of the base of an isosceles triangle is 60 cm. If its area is 1200 sq. metre, find the length of equal sides.

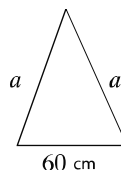
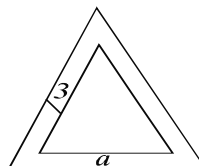
Solution : Let the base of the isosceles triangle be $b = 60$ cm. and the length of equal sides be a.

$\therefore$ Area of the triangle $= \dfrac{b}{4}\sqrt{4a^2 - b^2}$

According to the question, $\dfrac{b}{4}\sqrt{4a^2 - b^2} = 1200$

or, $\dfrac{60}{4}\sqrt{4a^2 - (60)^2} = 1200$

or, $15\sqrt{4a^2 - 3600} = 1200$

or, $\sqrt{4a^2 - 3600} = 80$

or, $4a^2 - 3600 = 6400$; [by squaring]
or, $4a^2 = 10000$
or, $a^2 = 2500$
$\therefore\ a = 50$

$\therefore$ The length of equal sides of the triangle is 50 cm.

Example 6. From a definite place two roads run in two directions making are angle 120^o. From that definite place, persons move in the two directions with speed of 10 km per hour and 8 km per hour respectively. What will be the direct distance between them after 5 hours ?

Solution : Let two men start from A with velocities 10 km/hour and 8 km/hour respectively and reach B and C after 5 hours. Then after 5 hours, the direct distance between them is BC. From C perpendicular CD is drawn on BA produced.

$\therefore\ AB = 5\times 10$ km = 50 km, $\quad AC = 5\times 8$ km = 40 km.

and $\angle BAC = 120^o$

$\therefore\ \angle DAC = 180° - 120° = 60°$

From the right angled triangle ACD

$\therefore\ \dfrac{CD}{AC} = \sin 60^o$ or, $CD = AC\sin 60^o = 40\times\dfrac{\sqrt{3}}{2} = 20\sqrt{3}$

and $\dfrac{AD}{AC} = \cos 60^o$ or, $AD = AC\cos 60^o = 40\times\dfrac{1}{2} = 20$

Again, we get from right angled ΔBCD,

$BC^2 = BD^2 + CD^2 = (BA + AD)^2 + CD^2$
$\quad = (50+20)^2 + (20\sqrt{3})^2 = 4900 + 1200 = 6100$

$\therefore\ BC = 78\cdot 1$ (app.)

Required distance is $78\cdot 1$ km. (approx)

Example 7. The lengths of the sides of a triangle are 25, 20, 15 units respectively. Find the areas of the triangles in which it is divided by the perpendicular drawn from the vertex opposite of the greatest side.

Solution : Let in triangle ABC, $BC = 25$ units, $AC = 20$ units, $AB = 15$ units.

The drawn perpendicular AD from vertex A on side BC divides the triangular region into ΔABD and ΔACD.

Let $BD = x$ and $AD = h$

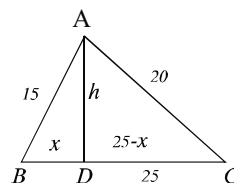
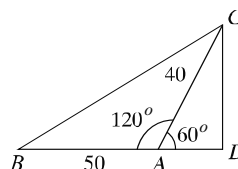
$\therefore\ CD = BC - BD = 25 - x$

In right angle ΔABD

$BD^2 + AD^2 = AB^2$ or, $x^2 + h^2 = (15)^2$

$\therefore\ x^2 + h^2 = 225$..........$(i)$

and ΔACD is right angled

$CD^2 + AD^2 = AC^2$ or, $(25-x)^2 + h^2 = (20)^2$

or, $625 - 50x + x^2 + h^2 = 400$
or, $625 - 50x + 225 = 400$; [with the help of equation (i)]
or, $50x = 450$; $\therefore$ $x = 9$
Putting the value of x in equation (i), we get,
$81 + h^2 = 225$ or, $h^2 = 144$ $\therefore$ $h = 12$
Area of $\Delta\ ABD = \frac{1}{2} BD \cdot AD = \frac{1}{2} \times 9 \times 12$ square units $= 36$ square units
and area of $\Delta\ ACD = \frac{1}{2} BD \cdot AD = \frac{1}{2}(25 - 9) \times 12$ square units
$= \frac{1}{2} \times 16 \times 12$ square units $= 96$ square units
Required area is 36 square units and 96 square units.

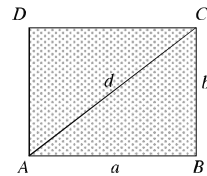
Exercise 16·1

1. The hypotenuse of a right angled triangle is 25 m. If one of its sides is $\frac{3}{4}$th of the other, find the length of the two sides.
2. A ladder with length 20m. stands vertically against a wall. How much further should the lower end of the end of the ladder be moved so that its upper end descends 4 metre?
3. The perimeter of an isosceles triangle is 16 m. If the length of equal sides is $\frac{5}{6}$th of base, find the area of the triangle.
4. The lengths of the two sides of a triangle are 25 cm., 27 cm. and perimeter is 84 cm. Find the area of the triangle.
5. When the length of each side of an equilateral triangle is increased by 2 metre, its area is increased by $6\sqrt{3}$ square metre. Find the length of side of the triangle.
6. The lengths of the two sides of a triangle are 26 m., 28 m. respectively and its area is 182 square metre. Find the angle between the two sides.
7. The perpendicular of a right angled triangle is 6cm less than $\frac{11}{12}$ times of the base, and the hypotenuse is 3 cm less than $\frac{4}{3}$ times of the base. Find the length of the base of the triangle.
8. The length of equal sides of an isosceles triangle is 10m and area 48 square metre. Find the length of the base.

9. Two roads run from a definite place with an angle of 135^o in two dinrections. Two persons move from the definite place in two directions with the speed of 7 km per hour and 5 km per hour respectiv ely. What will be the direct distance between them after 4 hours?
10. If the lengths of the perpendiculars from a point interior of an equilateral triangle to three sides are 6 cm., 7 cm., 8 cm. respectively;find the length of sides of the triangle and the area of the triangular region.

16·2 Area of Quadrilateral Region

(1) Area of rectangular region

Let, the length of $AB = a$, breadth $BC = b$ and diagonal $AC = d$, of rectangle $ABCD$. We know, the diagonal of a rectangle divides. the rectangle into two equal triangular regions.

$\therefore$ Area of the rectangle $ABCD = 2\times$ area of $\Delta ABC = 2\times\frac{1}{2}a\cdot b = ab$

perimeter of the rectangular region, $s = 2(a+b)$

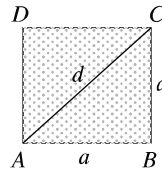
the ΔABC is right angled.

$AC^2 = AB^2 + BC^2$ or, $d^2 = a^2 + b^2$; $\therefore$ $d = \sqrt{a^2+b^2}$

(1) Area of square region

Let the length of each side of a square $ABCD$ be *a* and diagonal *d*. The diagonal AC divides the square region into two equal triangular regions.

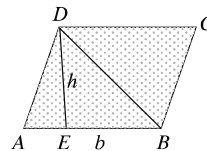
$\therefore$ Area of square region $ABCD = 2\times$ area of Δ ABC

$= 2\times\frac{1}{2}a\cdot a = a^2$

Observe that, the perimeter of the square region $s = 4a$

and diagonal $d = \sqrt{a^2+a^2} = \sqrt{2a^2} = \sqrt{2}a$

(3) Area of parallelogram region.

(a) Base and height are given.

Let, the base $AB = b$ and height $DE = h$ of parallelogram $ABCD$ The diagonal BD divides the parallelogram into two equal triangular regions.

$\therefore$ The area of parallelogram $ABCD = 2\times$ area of Δ ABD

$= 2\times\frac{1}{2}b\cdot h$

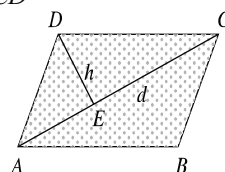
$= bh$

(b) The length of a diagonal and the length of a perpendicular drawn from the opposite angular point on that diagonal are given.

Let, the diagonal be $AC = d$ and the perpendicular from opposite angular point D on AC be $DE = h$ of a parallelogram $ABCD$. Diagonal AC divides the parallelogram into two equal triangular regions.

$\therefore$ The area of parallelogram region $ABCD$ = $2\times$ area of Δ ACD

$$= 2\times\frac{1}{2}d\cdot h$$

$$= dh$$

(4) Area of Rhombus Region

Two diagonals of a rhombus region are given

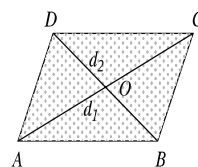
Let the diagonals be $AC = d_1$, $BD = d_2$ of the rhombus $ABCD$ and the diagonals intersect each other at O. Diagonal AC divides the rhombus region into two equal triangular regions.

We know that the diagonals of a rhombus bisect each other at right angles.

$\therefore$ Height of $\Delta ACD = \frac{d_2}{2}$

$\therefore$ Area of the rhombus region $ABCD$ = $2\times$ area of Δ ACD

$$= 2\times\frac{1}{2}d_1\times\frac{d_2}{2}$$

$$= \frac{1}{2}d_1d_2$$

(5) Area of trapezium region

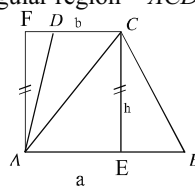
Two parallel sides of trapezium region and the distance of perpendicular between them are given.

Let $ABCD$ be a trapezium whose lengths of parallel sides are $AB = a$ unit, $CD = b$ unit and distance between them be $CE = h$. Diagonal AC divides the trapezium region $ABCD$ into ΔABC and ΔACD.

Area of trapezium region $ABCD$

= Area of the triangular region ABC + Area of the triangular region ACD.

$$= \frac{1}{2}AB\times CE + \frac{1}{2}CD\times AF$$

$$= \left(\frac{1}{2}ah + \frac{1}{2}bh\right) = \frac{1}{2}h(a+b)$$

Example 1. Length of a rectangular room is $\frac{3}{2}$ times of breadth. If the area is 384 square metre, find the perimeter and length of the diagonal.

Solution : Let breath of the rectangular room is x metre

$\therefore$ Length of the room is $\frac{3x}{2}$ metre

$\therefore$ Area $\frac{3x}{2}\times x$ or, $\frac{3x^2}{2}$ square metre.

According to the question, $\frac{3x^2}{2}=384$ or, $3x^2=768$ or, $x^2=256$; $\therefore$ $x=16$ metre

$\therefore$ Length of the rectangular room = $\frac{3}{2}\times 16$ metre = 24 metre

and breath = 16 metre.

$\therefore$ Its perimeter = 2 (24 + 16) metre = 80 metre.

and length of the diagonal = $\sqrt{(24)^2+(16)^2}$ metre = $\sqrt{832}$ metre = $28\cdot 84$ metre (app.)

The required perimetre is 80 metre and length of diagonal is $28\cdot 84$ metre (approx)

Example 2. The area of a rectangular region is 2000 square metre. If the length is reduced by 10 metre, it becomes a square region. Find the length and breadth of the rectangular region.

Solution : Let length of the rectangular region be x metre and breadth y metre.

$\therefore$ Area of the rectangular region = xy square metre

According to the question $xy=2000$................(1)

and $x-10=y$............(2)

We get, from equation (2) $y=x-10$............(3)

From equations (1) and (3) we get,

$x(x-10)=2000$ or, $x^2-10x-2000=0$

or, $x^2-50x+40x-2000=0$ or, $(x-50)(x+40)=0$

$\therefore$ $x-50=0$ or, $x+40=0$

or, $x=50$ or, $x=-40$

But length can never be negative.

$\therefore$ $x=50$

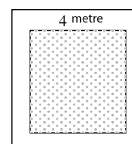
Now puting the value of x in equation (3) we get

$y=50-10=40$

$\therefore$ length = 50 m. and breadth = 40 m.

Example 3. There is a road of 4 metre width inside around a square field. If the area of the road is 1 hector, determine the area of the field excluding the road.

Solution : Let, the length of the square field is x metre.

$\therefore$ Its area is x^2 square metre.

There is a road around the field with width 4 m.

$\therefore$ Length of the square field excluding the road = $(x - 2 \times 4)$, or $(x - 8)$ m

$\therefore$ Area of the square field excluding the road is $(x - 8)^2$ square m.

Area of the road = $\{x^2 - (x - 8)^2\}$ square m.

We know, 1 hector = 10000 square m.

According the question, $x^2 - (x - 8)^2 = 10000$

or, $x^2 - x^2 + 16x - 64 = 10000$

or, $16x = 10064$

$\therefore \quad x = 629$

Area of the square field excluding the road = $(629 - 8)^2$ square m.

= 385641 square m.

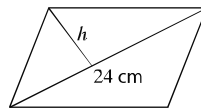
= $38 \cdot 56$ hector (approx.)

The required area is $38 \cdot 56$ hector (approx.)

Example 4. The area of a parallelogram is 120 sq. cm. and length of one of its diagonal is 24 cm. Determine the length of the perpendicular drawn on that diagonal from the opposite vertex.

Solution : Let a diagonal of a parallelogram be $d = 24$ cm. and the length of the perpendicular drawn on the diagonal from the opposite vertex be h cm.

$\therefore$ Area of the parallelogram = dh square cm.

As per question, $dh = 120$ or, $h = \frac{120}{d} = \frac{120}{24} = 5$

The required length of the perpendicular is 5 cm.

Example 5. If the length of the sides of a parallelogram are 12 m. 8 m. If the length of the smaller diagonal is 10 m, determine the length of the other diagonal.

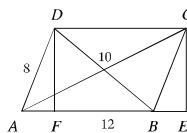
Solution : Let, in the parallelogram $ABCD$; $AB = a = 12$ m. and $AD = c = 8$ m. and diagonal $BD = b = 10$ m. Let us draw the perpendiculars DF and CE from D and C on the extended part of AB, respectively. Join A, C and B, D.

$\therefore$ Semi perrimeter of ΔABD is $s = \frac{12 + 10 + 8}{2}$ m. = 15 m.

$\therefore$ Area of the triangular region ABD = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{15(15-12)(15-10)(15-8)}$ sq. m.

= $\sqrt{1575}$ sq. m.

= $39 \cdot 68$ sq. m. (approx.)

Again, area of the triangular region ABD = $\frac{1}{2} AB \times DF$

or, $39 \cdot 68 = \frac{1}{2} \times 12 \times DF$ or, $6 \cdot DF = 39 \cdot 68$; $\therefore$ $DF = 6 \cdot 61$

Now, in right angled triangle ΔBCE,

$$BE^2 = BC^2 - CE^2 = AD^2 - DF^2 = 8^2 - (6\cdot 61)^2 = 20\cdot 31$$

$\therefore\ BE = 4\cdot 5$

Therefore, $AE = AB + BE = 12 + 4\cdot 5 = 16\cdot 5$

From right angled triangle ΔBCE, we get,

$AC^2 = AE^2 - CE^2 = (16\cdot 5)^2 - (6\cdot 61)^2 = 315\cdot 94$

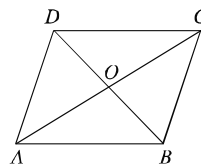
$\therefore\ AC = 17\cdot 77$ (approx.)

$\therefore$ The required length of the diagonal is $17\cdot 77$ m. (approx.)

Example 6. The length of a diagonal of a rhombus is 10m. and its area is 120 sq. m. Determine the length of the other diagonal and its perimeter.

Solution : Let, the length of a diagonal of rhombus $ABCD$ is $BD = d_1 = 10$ metre and another diagonal $AC = d_2$ metre.

$\therefore$ Area of the rhombus = $\frac{1}{2}d_1d_2$ sq. m.

As per question, $\frac{1}{2}d_1d_2 = 120$ or, $d_2 = \frac{120\times 2}{10} = \frac{120\times 2}{10} = 24$

Now, the diagonals of rhombus bisect each other at right angles. Let the diagonals interset at the point 0.

$\therefore\ OD = OB = \frac{10}{2}$ m. = 5 m. and $OA = OC = \frac{24}{2}$ m. = 12 m.

and in right angled triangle ΔAOD, we get

$AD^2 = OA^2 + OD^2 = 5^2 + (12)^2 = 169$; $\therefore\ AD = 13$

$\therefore$ The length of each sides of the rhombus is 13 m.

$\therefore$ The perimeter of the rhombus = 4×13 m. = 52 m..

The required length of the diagonal is 24 m. and perimeter is 52 m.

Example 7. The lengths of two parallel sides of a trapezium are 91 cm. and 51 cm. and the lengths of two other sides are 37 cm and 13 cm respectively. Determine the area of the trapezium.

Solution : Let, in trapezium $ABCD$; $AB = 91$ cm. $CD = 51$ cm. Let us draw the perpendiculars DF and CF on AB from D and C respectively.

$\therefore\ CDEF$ is a rectangle.

$\therefore\ EF = CD = 51$ cm.

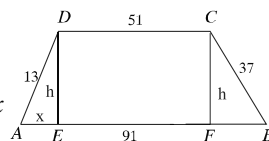
Let, $AE = x$ and $DE = CF = h$

$\therefore\ BF = AB - AF = 91 - (AE + EF) = 91 - (x + 51) = 40 - x$

$\therefore$ From right angled triangle ΔADE, we get,

$AE^2 + DE^2 = AD^2$ or, $x^2 + h^2 = (13)^2$ or, $x^2 + h^2 = 169$.........(i)

Again, from right angled triangle ΔBCF, we get,

$BF^2 + CF^2 = BC^2$ or, $(40 - x)^2 + h^2 = (37)^2$

or, $1600-80x+x^2+h^2=1369$
or, $1600-80x+169=1396$ [with the help of (1)]
or, $1600+169-1396=80x$;or, $80x=400$; $\therefore\ x=5$
Now putting the value of x in equation (1) we get
$5^2+h^2=163$ or, $h=169-25=144$; $\therefore\ h=12$

Area of $ABCD$ = $\frac{1}{2}(AB+CD)\cdot h$

= $\frac{1}{2}(91+51)\times 12$ square cm.

= 852 square cm.

The required area is 852 square cm.

16·3 Area of regular polygon

The lengths of all sides of a regular polygon are equal. Again, the angles are also equal. Regular polygon with n sides produces n isocsceles triangles by adding centre to the vertices.

$\therefore$ Area of the regular polygon = $n\times$ area of one triangular region.

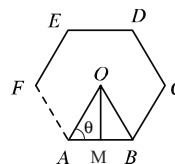
Let $ABCDEF$ be a regular polygon whose centre is O.
$\therefore$ It has n sides and the length of each side is a. We join *O, A ; O, B*.
$\therefore$ Let in ΔAOB height $OM=h$ and $\angle OAB=\theta$
$\therefore$ The angle produced at each of the vertices of regular polygon = 2θ .
$\therefore$ Angle produced by n number of vertices in the polygon = $2\theta\cdot n$
Angle produced in the polygon at the centre = 4 right angles.
The sum of angles of n number of triangles = $2\theta(n+4)$ right angles.
$\therefore$ Sum of 3 angles of ΔOAB = 2 right angles.
$\therefore$ The wise, summation of the angles of n numbers of triangles = $n\cdot 2$ right angles
$\therefore$ $2\theta(n+4)$ right angles = $n\cdot 2$ right angles

or, $2\theta\cdot n=(2n-4)$ right angles

or, $\theta=\frac{2n-4}{2n}$ right angles

or, $\theta=\left(1-\frac{2}{n}\right)$ right angles

or, $\theta=\left(1-\frac{2}{n}\right)\times 90^\circ=90^\circ-\frac{180^\circ}{\text{n}}$

Now, $\tan\theta=\frac{h}{\frac{a}{2}}=\frac{2h}{a}$; $\therefore\ h=\frac{a}{2}\tan\theta$

$\therefore$ Area of $\Delta OAB = \frac{1}{2}ah$

$= \frac{1}{2}a \times \frac{a}{2}\tan\theta$

$= \frac{a^2}{4}\tan\left(90^\circ - \frac{180^\circ}{n}\right)$

$= \frac{a^2}{4}\cot\left(\frac{180^\circ}{n}\right)$

$\therefore$ Area of a regular polygon having n sides $= \frac{a^2}{4}\cot\left(\frac{180^\circ}{n}\right)$.

Example 8. If the length of each side of a regular pentagon is 4 cm, determine its area.

Solution : Let, length of each side of a regular pentagon is $a = 4$ cm.

and number of sides $n = 5$

We know, area of a regular polygon $= \frac{a^2}{4}\cot\frac{180^\circ}{n}$

$\therefore$ Area of the pentagon $= \frac{4^2}{4}\cot\frac{180^\circ}{5}$ sq. cm.

$= 4 \times \cot 36^o$ sq. cm.

$= 4 \times 1\cdot376$ sq. cm. [with the help of calculator]

$= 5\cdot506$ sq. cm. (approx.)

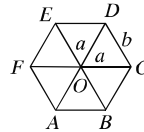
The required area $= 5\cdot506$ sq. cm. (approx.)

Example 9. The distance of the centre to the vertex of a regular hexagon is 4 m. Determine its area.

Solution : Let, $ABCDEF$ is a regular hexagon whose centre is O, O is joined to each of the vertex and thus 6 triangles of equal area are formed.

$\therefore \angle COD = \frac{360^\circ}{6} = 60^\circ$

Let the distance of centre O to its vertex is a m.

$\therefore$ a = 4.

$\therefore$ Area of $\Delta COD = \frac{1}{2}a \cdot a\sin 60^\circ = \frac{1}{2}a^2 \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}a^2$

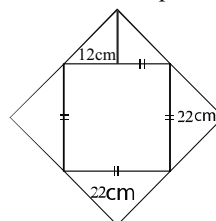
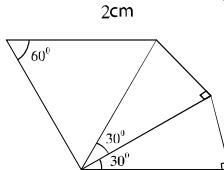
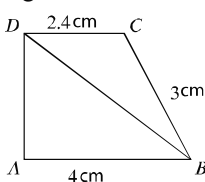
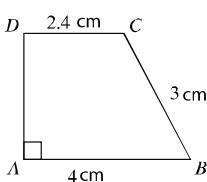
$= \frac{\sqrt{3}}{4} \times 4^2$ sq. m. $= 4\sqrt{3}$ sq. m.

$\therefore$ **Area of the regular hexagon**

$= 6 \times 4\sqrt{3}$ sq, m.
$= 24\sqrt{3}$ sq. m.

Exercise 16·2

1. The length of a rectangular region is twice its width. If its area is 512 sq. m., determine its perimeter.
2. The length of a plot is 80 m. and the breadth is 60 m. A rectangular pond was excavated in the plot. If the width of each side of the border around the pond is 4 metre, determine the area of the border of the pond.
3. The length of a garden is 40 metre and its breadth is 30 metre. There is a pond in side the garden with around border of equal width. If the area of the pond is $\frac{1}{2}$ of that of the garden, find the length and breadth of the pond.
4. Outside a square garden, there is a path 5 metre width around it. If the area of the path is 500 square metre, find the area of the garden.
5. The perimeter of a square region is equal to the perimeter of a rectangular region. The length of the rectangular region is thrice its breadth and the area is 768 sq. m. How many stones will be required to cover the square region with square stones of 40 cm each?
6. Area of a rectangular region is 160 sq. m. If the length is reduced by 6 m., it becomes a square region. Determine the length and the breadth of the rectangle.
7. The base of a parallelogram is $\frac{3}{4}$th of the height and area is 363 square inches. Determine the base and the height of the parallelogram.
8. The area of a parallelogram is equal to the area of a square region. If the base of the parallelogram is 125m. and the height is 5 m, find the length of the diagonal the square.
9. The length of two sides of a parallelogram are 30 cm and 26 cm, If the smaller diagonal is 28 cm, find the length of the other diagonal.
10. The perimeter of a rhombus is 180 cm. and the smaller diagonal is 54 cm. Find the other diagonal and the area.
11. Deference of the length of two parallel sides of a trapezium is 8 cm. and their perpendicular distance is 24 cm. Find the lengths of the two parallel sides of the trapezium.
12. The lengths of two parallel sides of a trapezium are 31 cm. and 11 cm. respectively and two other sides are 10 and 12 cm. respectively. Find the area of the trapezium.

13. The distance from the centre to the vertex of a regular octagon is 1.5 m. Find the area of the regular octagon.
14. The length of a rectangular flower garden is 150 m. and breadth is 100 m. For nursing the garden, there is a path with 3 m. width all along its length and breadth right at the middle of the garden.
    (a) Describe the above information with figure.
    (b) Determine the area of the path.
    (c) How many bricks of 25 cm. length and 12 ·5 cm. width will be required to make the path metalled ?
15. From the figure of the polygon determine its area.

16. From the information given below determine the area of the figures :

6.4 Measurement regarding circle

(1) Circumference of a Circle

The length of a circle is called its circumference. Let r be the radius of a circle,

its circumference $c = 2\pi\, r$, where $\pi = 3\cdot14159265$.......... which is an irrational number value of $\pi = 3\cdot1416$ is used as the actual value.

Therefore, if the radius of a circle is known, we can find the approximate value of the circumference of the circle by using the value of π.

Example 1. The diameter of a circle is 26 cm. Find its circumference.

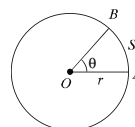
Solution : Let, the radius of the circle is r.

$\therefore$ diameter of the circle = $2r$ and circumference = $2\pi\, r$

As per question, $2r = 26$ or, $r = \frac{26}{2}$ $\quad\therefore\ r = 13$

$\therefore$ circumference of the circle = $2\pi\, r = 2\times3\cdot1416\times13$ cm.

$= 3\cdot1416\times26$ cm (approx.)

The required circumference of the circle is $81\cdot68$ cm. (approx.)

(2) Length of arc of a circle

Let O be the centre of a circle whose radius is r and are $AB = s$, which produces θ^o angle at the centre.

$\therefore$ Circumference of the circle = $2\pi r$

Total angle produced at the centre of the circle = 360^o and arc s produces angle $\theta°$ at the centre. We know, any interior angle at the centre of a circle produced by any arc is proportional to the arc.

$\therefore \dfrac{\theta}{360} = \dfrac{s}{2\pi r}$ or, $s = \dfrac{\pi r\theta}{180}$

(3) Area of circular region and circular segment

The subset of the plane formed by the union of a circle and its interior is called a circular region and the circle is called the boundary of the such circular region.

Circular segment: The area formed by an arc and the radius related to the joining points of that arc is called circular segment.

If A and B are two points on a circle with centre O, the subset of the plane formed by the union of the intersection of $\angle AOB$ and the interior of the circle with the line segment OA, OB and the arc AB, is called a circular segment.

In previous class, we have learnt that if the radius of a circle is r, the area is = πr^2

We know, any angle produced by an arc at the centre of a circle is proportional to the arc.

So, at this stage we can accept that the area of two circular segments of the same circle are proportional to the two arcs on which they stand.

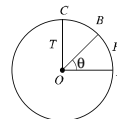
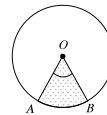
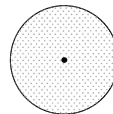
Let us draw a radius r with centre O.

The circular segment AOB stands on the arc APB whose measurement is θ. Draw a perpendicular OC on OA.

$$\therefore \frac{\text{Area of circular segment } AOB}{\text{Area of circular segment } AOC} = \frac{\text{Measurement of } \angle AOB}{\text{Measurement of } \angle AOC}$$

$$\text{or, } \frac{\text{Area of circular segment } AOB}{\text{Area of circular segment } AOC} = \frac{\theta}{90}\ ;\ [\angle AOC = 90^o]$$

$$\text{or, Area of circular segment } AOB = \frac{\theta}{90} \times \text{ area of circular segment } AOC$$

$$= \frac{\theta}{90} \times \frac{1}{4} \times \text{ area of the circle}$$

$$= \frac{\theta}{90} \times \frac{1}{4} \times \pi r^2$$

$$= \frac{\theta}{360} \times \pi r^2$$

So, area of circular segment $= \dfrac{\theta}{360} \times \pi r^2$

Example 2. The radius of a circle is 8 cm. and a circular segment substends an angle 56^o at the centre. Find the length of the arc and area of the circular segment.

Solution : Let, radius of the circle, $r = 8$ cm, length of arc is s and the angle subtended by the circular segment is 56^o.

We know, $s = \dfrac{\pi r \theta}{180} = \dfrac{3 \cdot 1416 \times 8 \times 56}{180}$ cm.. $= 7 \cdot 82$ cm. (approx)

Area of circular segment $= \dfrac{\theta}{360} \times \pi r^2$

$= \dfrac{56}{360} \times 3.1416 \times 8^2$ sq.cm.

$= 62 \cdot 55$ sq. cm. (approx)

Example 3. If the difference between the radius and circumference of a circle is 90 cm., find the radius of the circle.

Solution : Let the radius of the circle be r

$\therefore$ Diameter of the circler is $2r$ and circumference $= 2\pi r$

As per question, $2\pi r - 2r = 90$

or, $2r(\pi - 1) = 90$ or, $r = \dfrac{90}{2(\pi - 1)} = \dfrac{45}{3 \cdot 1416 - 1} = 21 \cdot 01$ (approx.)

The required radius of the circle is $21 \cdot 01$ cm. (approx.).

Example 4. The diameter of a circular field is 124 m. There is a path with 6 m. width all around the field. Find the area of the path.

Solution : Let the radius of the circular field be r and radius of the field with the path be R.

$\therefore$ $r = \dfrac{124}{2}$ m. $= 62$ m. and $R = (62 + 6)$ m. $= 68$ m.

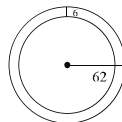
Area of the circular field = πr^2

and area of the circular field with the path = πR^2

$\therefore$ Area of the path = Area of field with path – Area of the field

$= (\pi R^2 - \pi r^2) = \pi (R^2 - r^2)$

$= 3 \cdot 1416\{(68)^2 - (62)^2\}$ sq. m.

$= 3 \cdot 1416(4624 - 3844)$

$= 3 \cdot 1416 \times 780$ sq. m.

$= 2450 \cdot 45$ sq. m. (approx)

The required area of the path is $2450 \cdot 44$ square m. (approx.)

Activity : Circumference of a circle is 440 m. Determine the length of the sides of the inscribed square in it.

Example 5. The radius of a circle is 12 cm. and the length of an arc is 14 cm. Determine the angle subtended by the circular segment at its centre.

Solution : Let, radius of the circle is r = 12 cm., the length of the arc is s = 14 cm. and the angle subtended at the centre is θ °.

We know, $s = \dfrac{\pi s \theta}{180}$

or, $\pi r \theta = 180 \times s$

or, $\theta = \dfrac{180 \times s}{\pi r} = \dfrac{180 \times 14}{3 \cdot 1416 \times 12} = 66 \cdot 85$ (approx)

$\therefore$ The required angle is 66·85° (approx)

Example 6. Diameter of a wheel is 4.5 m. for traversing a distance of 360 m.;how many times the wheel will revolve ?

Solution : Given that, the diameter of the wheel is 4.5 m.

$\therefore$ The radius of the wheel, $r = \dfrac{4 \cdot 5}{2}$ m. and circumference = $2\pi r$

Let, for traversing 360 m, the wheel will revolve n times

As per question, $n \times 2\pi r = 360$

or, $n = \dfrac{360}{2\pi r} = \dfrac{360 \times 2}{2 \times 3 \cdot 1416 \times 4 \cdot 5}$ = 25.46 (approx)

$\therefore$ The wheel will revolve 25 times (approx) for traversing 360 m..

Example 7. Two wheels revolve 32 and 48 times respectively to cover a distance of 211 m. 20 cm. Determine the difference of their radii.

Solution : 211 m. 20 cm. = 21120 cm.

Let, the radii of two wheels are R and r respectively ;where $R > r$.

$\therefore$ Circumferences of two wheels are $2\pi R$ and $2\pi r$ respectively and the difference of radii is $(R - r)$

As per question, $32 \times 2\pi R = 21120$

or, $R = \dfrac{21120}{32 \times 2\pi} = \dfrac{21120}{32 \times 2 \times 3 \cdot 1416} = 105 \cdot 04$ (approx)

and $48 \times 2\pi r = 21120$

or, $r = \dfrac{21120}{48 \times 2\pi} = \dfrac{21120}{48 \times 2 \times 3 \cdot 1416} = 70 \cdot 03$ (approx)

$\therefore\ R - r = (105\cdot04 - 70\cdot03)$ cm.. = $35\cdot01$ cm. = $\cdot35$ m. (approx)

$\therefore$ The difference of radii of the two wheels is $\cdot35$ m (approx)

Example 8. The radius of a circle is 14 cm. The area of a square is equal to the area of the circle. Determine the length of the square.

Solution : Let the radius of the circle, $r = 14$ cm. and the length of the square is a

$\therefore$ Area of the square region = a^2 and the area of the circle = πr^2

According to the question, $a^2 = \pi r^2$

$\therefore$ Radius of the half circle $r = \dfrac{22}{2}$ m. = 11 m.

or, $a = \sqrt{\pi}r = \sqrt{3\cdot1416} \times 14 = 24\cdot81$ (approx)

The required length is 24·81 cm. (approx.)

Example 9 : In the figure, $ABCD$ is a square whose length of each side is 22 m. and AED region is a half circle. Determine the area of the whole region.

Solution : Let, the length of each side of the square $ABCD$ be a.

$\therefore$ Area of square region = a^2

Again, AED is a half circle.

Let ;be its radius

$\therefore$ Area of the half circle, $AED = \dfrac{1}{2}\pi r^2$

$\therefore$ Area of the whole region = Area of the square $ABCD$ + area of the half circle AED.

$= a^2 + \dfrac{1}{2}\pi r^2$

$= \{(22)^2 + \dfrac{1}{2} \times 3\cdot1416 \times (11)^2\}$ sq. metre

[a = 22, r = $\dfrac{22}{2}$ = 11]

= 674.07 sq. m (app.)

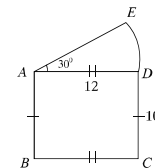
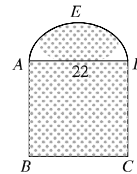
The required area is 674.07 square metre (approx)

Example 10. In the figure, $ABCD$ is a rectangle whose the length is 12 m., the breadth is 10 m. and DAE is a circular region.

Determine the length of the arc DE and the area of the whole region.

Solution : Let the radius of the circular segment, $r = AD = 12$ m. and the angle subtended at centre $\theta = 30^\circ$

$\therefore$ length of the arc $DE = \dfrac{\pi r\theta}{180}$

$= \dfrac{3\cdot1416 \times 12 \times 30}{180}$ m. = 6.28 m. (approx.)

$\therefore$ Area of the circular segment DAE = $\frac{\theta}{360}\times\pi r^2 = \frac{30}{360}\times 3\cdot1416\times(12)^2$ sq. m. = 37.7 sq.cm (approx)

The length of the rectangle $ABCD$ is 12 m. and the breadth is 10 m.

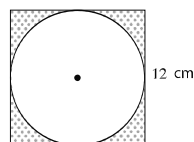
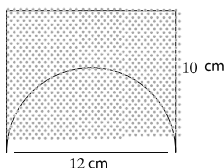
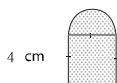
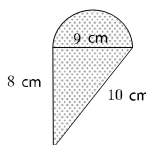
$\therefore$ Area of the rectangle = length × breadth = 12 × 10 sq. m = 120 sq. m.

$\therefore$ Area of the whole region = (37.7 + 120) sq. m. = 157.7 square metre.

The required area is 157.7 square metre. (approx)

Activity : Determine the area of the dark marked region in the figure.

Exercise 16.3

1. Angle subtended by a circular segment at the centre is 30°. If the diameter of the circle is 126 cm., determine the length of the arc.
2. A horse turned around a circular field with a speed of 66 m per minute in $1\frac{1}{2}$ minute. Determine the diameter of the field.
3. Area of a circular segment is 77 sq. m. and the radius is 21 m. Determine the angle subtended at the centre by the circular segment.
4. The radius of a circle is 14 cm. and an arc subtends an angle 76° at its centre. Determine the area of the circular segment.
5. There is a road around a circular field. The outer circumference of the road is greater than the inner circumference by 44 metres. Find the width of the road.
6. The diameter of a circular park is 26 m. There is a road of 2 m. width around the park outside. Determine the area of the road.
7. The diameter of the front wheel of a car is 28 cm. and the back wheel is 35 cm. To cover a distance of 88 m, how many times more the front wheel will revolve than the back one ?
8. The circumference of a circle is 220 m. Determine the length of the side of the inscribed square in the circle.
9. The circumference of a circle is equal to the perimeter of an equilateral triangle. Determine the ratio of their areas ?
10. Determine the area of the dark marked region with the help of the information given below :

6·5 Rectangular solid

The region surrounded by three pairs of parallel rectangular planes is known as rectangular solid.

Let, $ABCDEFGH$ is a rectangular solid, whose length $AB = a$, and breadth $BC = b$ and height $AH = c$

(1) Determining the diagonal: AF is the diagonal of the rectangular solid $ABCDEFGH$

In ΔABC, $BC \perp AB$ and AC is hypotenuse

$\therefore\ AC^2 = AB^2 + BC^2 = a^2 + b^2$

Again, in ΔACF, $FC \perp AC$ and AF is hypotenuse

$\therefore\ AF^2 = AC^2 + CF^2 = a^2 + b^2 + c^2$

$\therefore\ AF = \sqrt{a^2 + b^2 + c^2}$

$\therefore$ the diagonal of the rectangular solid $= \sqrt{a^2 + b^2 + c^2}$

(2) Determination of area of the whole surface :

There are 6 surfaces of the rectangular solid where the opposite surfaces are equal figure

Area of the whole surface of the rectangular solid

= 2 (area of the surface of $ABCD$ + area of the surface of $ABGH$ + area of the surface of $BCFG$)

$= 2(AB \times AD + AB \times AH + BC \times BG)$

$= 2(ab + ac + bc)$

$= 2(ab + bc + ca)$

(3) Volume of the rectangular solid = length $\times$ width $\times$ height

$= abc$

Example 1. The length, width and height of a rectangular solid are 25 cm., 20 cm. and 15 cm. respectively. Determine its area of the whole surface, volume and the length of the diagonal.

Solution : Let, the length of the rectangular solid is $a = 25$ cm., width $b = 20$ cm. and height $c = 15$ cm.

$\therefore$ Area of the whole surface of the rectangular solid $= 2(ab + bc + ca)$

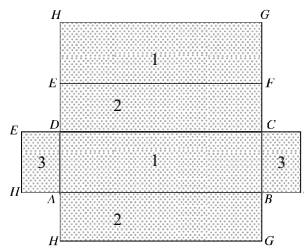
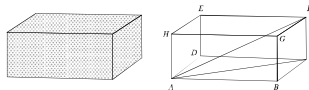
$= 2(25 \times 20 + 20 \times 15 + 15 \times 25)$ sq. cm.

$= 2350$ square cm.

Volume $= abc$

$= 25 \times 20 \times 15$ cube cm.

$= 7500$ cube cm.

And the length of its diagonal $= \sqrt{a^2 + b^2 + c^2}$

$= \sqrt{(25)^2 + (20)^2 + (15)^2}$ cm.

$= \sqrt{625 + 400 + 225}$ cm.

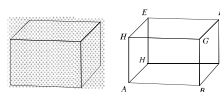
$= \sqrt{1250}$ cm.

$= 35 \cdot 353$ cm. (approx.)

The required area of the whole surface is 2350 cm^2., volume 7500 cm^3. and the length of the diagonal is $35 \cdot 353$ cm. (approx.).

Activity : Determine the volume, area of the whole surface and the length of the diagonal of your mathematics book calculating its length, width and height.

6·6 Cube

If the length, width and height of a rectangular solid are equal, it is called a cube.

Let, $ABCDEFGH$ is a cube.

Its height = width = height = a

(1) The length of diagonal of the cube = $\sqrt{a^2 + a^2 + a^2} = \sqrt{3a^2} = \sqrt{3}a$

(2) The area of the whole surface of the cube $= 2(a \cdot a + a \cdot a + a \cdot a)$

$= 2(a^2 + a^2 + a^2) = 6a^2$

(3) The volume of the cube = $a \cdot a \cdot a = a^3$

Example 2. The area of the whole surface of a cube is 96 m^2. Determine the length of its diagonal.

Solution : Let, the sides of the cube is a

$\therefore$ The area of its whole surface $= 6a^2$ and the length of diagonal = $\sqrt{3}a$

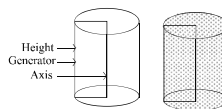
As per question, $6a^2 = 96$ or, $a^2 = 16$; $\therefore a = 4$

$\therefore$ The length of diagonal of the cube = $\sqrt{3}a = \sqrt{3} \times 4 = 6 \cdot 928$ m. (approx.)

The required length of the diagonal is $6 \cdot 928$ m. (approx.)

Activity : The sides of 3 metal cube are 3 cm., 4 cm. and 5 cm. respectively. A new cube is formed by melting the 3 cubes. Determine the area of the whole surface and the length of the diagonal of new cube.

6·7 Cylinder :

The solid formed by a complete revolution of any rectangle about one of its sides as axis is called a cylinder or a right circular cylinder. The two ends of a right circular cylinder are circles. The curved face is called curved surface and the total plane is called whole surface. The side of the rectangle which is parallel to the axis and revolves about the axis is called the generator line of the cylinder.

Let, figure (a) is a right circular cylinder, whose radius is r and height h

(1) Area of the base = πr^2

(2) Area of the curved surface

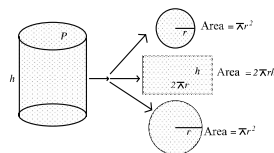
= perimeter of the base × height

$= 2\pi rh$

(3) Area of the whole surface

$= (\pi r^2 + 2\pi rh + \pi r^2) = 2\pi r(r+h)$

(4) The volume = area of the base × height

$= \pi r^2 h$

Example 3. If the height of a right circular cylinder is 10 cm. and radius of the base is 7 cm, determine its volume and the area of the whole surface.

Solution : Let, the height of the right circular cylinder is $h = 10$ cm. and radius of the base is r.

$\therefore$ Its volume $= \pi r^2 h = 3\cdot1416 \times 7^2 \times 10$

$= 1539\cdot38$ cube cm. (approx.)

And the area of the whole surface $= 2\pi r(r+h)$

$= 2 \times 3\cdot1416 \times 7(7+10)$ sq. cm. (approx.)

$= 747\cdot7$ sq. cm. (approx.)

Activity : Make a right circular cylinder using a rectangular paper. Determine the area of its whole surface and the volume.

Example 4. The outer measurements of a box with its top are 10 cm., 9 cm. and 7cm. respectively and the area of the whole inner surface is 262 cm^2. Find the thickness of its wall if it is uniform on all sides.

Solution : Let, the thickness of the box is x cm.

The outer measurements of the box with top are 10 cm., 9 cm. and 7 cm. respectively.

$\therefore$ The inside measurement of the box are respectively $a = (10-2x)$ cm., $b = (9-2x)$ cm. and $c = (7-2x)$ cm.

$\therefore$ The area of the whole surface of the inner side of the box $= 2(ab+bc+ca)$

As per question, $2(ab+bc+ca) = 262$

or, $(10-2x)(9-2x)+(9-2x)(7-2x)+(7-2x)(10-2x) = 131$

or, $90-38x+4x^2+63-32x+4x^2+70-34x+4x^2-131 = 0$

or, $12x^2-104x+92 = 0$

or, $3x^2-26x+23 = 0$

or, $3x^2-3x-23x+23 = 0$

or, $3x(x-1)-23(x-1) = 0$

or, $(x-1)(3x-23) = 0$

or, $x-1 = 0$ or, $3x-23 = 0$

or, $x = 1$ or, $x = \dfrac{23}{3} = 7\cdot67$ (approx.)

But the thickness of a box cannot be greater than or equal to the length or width or height

$\therefore\ x = 1$

The required thickness of the box is 1 cm.

Example 5. If the length of diagonal of the surface of a cube is $8\sqrt{2}$ cm., determine the length of its diagonal and volume.

Solution : Let, the side of the cube is a.

$\therefore$ The length of diagonal of the surface = $\sqrt{2}a$

Length of diagonal = $\sqrt{3}a$

And the volume $= a^3$

As per question, $\sqrt{2}a = 8\sqrt{2}$; $\therefore\ a = 8$

$\therefore$ The length of the cube's diagonal = $\sqrt{3} \times 8$ cm. = 8 $\sqrt{3}$ cm.

And the volume = 8^3 cm^3. = 512 cm^3.

The required length of the diagonal is 8 $\sqrt{3}$ cm. (approx) and the volume is 512 cm^3.

Example 6. The length of a rectangle is 12 cm. and width 5 cm. If it is revolved around the greater side, a solid is formed. Determine the area of its whole surface and the volume.

Solution : Given that, the length of a rectangle is 12 cm. and width 5 cm. If it is revolved around the greater side, a circle based right cylindrical solid is formed with height $h = 12$ cm. and radius of the base $r = 5$ cm.

$\therefore$ The whole surface of the produced solid = $2\pi r(r + h)$

$= 2 \times 3 \cdot 1416 \times 5(5 + 12)$ sq. sm.

$= 534 \cdot 071$ sq. cm. (approx.)

And the volume = $\pi r^2 h$

$= 3 \cdot 1416 \times 5^2 \times 12$ cm^3.

$= 942 \cdot 48$ cube cm^3. (approx.)

The required area of whole surface is $534 \cdot 071$ sq. cm. (approx.) and the volume is $942 \cdot 48$ cm^3. (approx.)

Exercise 16·4

1. The length and witdth of two adjacent sides of a parallelogram are 7 cm., and 5 cm. respectively. What is the half of its perimeter in cm.. ?

   (a) 12 (b) 20 (c) 24 (d) 28

2. The length of the side of an equilateral triangle is 6 cm. What is its area (cm^2) ?

   (a) $3\sqrt{3}$ (b) $4\sqrt{3}$ (c) $6\sqrt{3}$ (d) $9\sqrt{3}$

3. If the height of a trapezium is 8 cm. and the lengths of the parallel sides are 9 cm. and 7 cm. respectively, what is its area (cm^2) ?

   (a) 24 (b) 64 (c) 96 (d) 504

4. Follow the information given below :

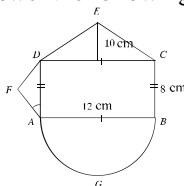
(i) A square stone with the side of 4 cm. has 16 cm. perimeter.

(ii) The area of circular sheet with the radius 3 cm.is 3π cm^2.

(iii) The volume of a cylinder with height of 5cm. and the radius of 2 cm. is 20π cm^3.

According to the information above, which one of the following is correct ?

(a) i and ii (b) i and iii (c) ii and iii (d) i, ii and iii

Answer the following questions (5 – 7) as per information from the picture below:

5. What is the length of the diagonal of the rectangle $ABCD$ in cm. ?

(a) 13 (b) 14 (c) $14 \cdot 4$ (app.) (d) 15

6. What is the area of the circular segment ADF in sq. cm.?

(a) 16 (b) 32 (c) 64 (d) 128

7. What is the circumference of the half circle AGB in cm. ?

(a) 18 (b) $18 \cdot 85$ (app.) (c) $37 \cdot 7$ (app.) (d) 96

8. The length, width and height of a rectangular solid are 16 m. 12 m. and $4 \cdot 5$ m. respectively. Determine the area of its whole surface, length of the diagonal and the volume.

9. The ratios of the length, width and height of a rectangular solid are $21:16:12$ and the length of diagonal is 87 cm. Determine the area of the whole surface of the solid.

10. A rectangular solid is standing on a base of area 48 m^2 Its height is 3m and diagonal is 13 m. Determine the length and width of the rectangular solid.

11. The outer measurements of a rectangular wooden box are 8 cm., 6 cm. and 4 cm., respectively and the area of the whole inner surface is 88 cm^2. Find the thickness of the wood of the box.

12. The length of a wall is 25 m, height is 6 m. and breadth is 30 cm. The length, breadth and height of a brick is 10 cm. 5 cm. and 3 cm. respectively. Determine the number of bricks to build the wall with the bricks.

13. The area of the surface of a cube is 2400 sq. cm. Determine the diagonal of the cube.

14. The radius and the height of a right circular cylinder are 12 cm. and 5 cm. respectively. Find the area of the curved surface and the volume of the cylinder.

15. The area of a curved surface of a right circular cylinder is 100 sq. cm. and its volume is 150 cubic cm. Find the height and the radius of the cylinder.

16. The area of the curved surface of a right circular cylinder is 4400 sq. cn. If its height is 30 cm., find the area of its whole surface.
17. The inner and outer diameter of a iron pipe is 12 cm. and 14 cm. respectively. If the height of the pipe is 5 m., find the weight of the iron pipe where weight of 7.2 gm. iron $= 1$ cm ^{3}.
18. The length and the breadth of a rectangular region are 12 m. and 5m. respectively. There is a circular region just around the rectangle. The places which is are not occupied by the rectangle, are planted with grass.
    (a) Describe the information above with a figure.
    (b) Find the diameter of the circular region.
    (c) If the cost of planting grass per sq. m. is Tk. 50, find the total cost.
19. ΔABC and ΔBCD are on the same base BC and on the same parallel lines BC and AD.
    (a) Draw a figure as per the description above.
    (b) Proof that Δ region ABC = Δ region BCD.
    (c) Draw a parallelogram whose area is equal to the area of ΔABC and whose one of the angles is equal to a given angle (construction and description of construction is must).
20. $ABCD$ is a parallelogram and $BCEF$ is a rectangle and BC is the base of both of them.
    (a) Draw a figure of the rectangle and the parallelogram assuming the same height.
    (b) Show that the perimeter of $ABCD$ is greater than the perimeter of $BCEF$.
    (c) Ratio of length and width of the rectangle is 5 : 3 and its perimeter is 48 m. Determine the area of the parallelogram